
4. A researcher measured the foot lengths of a random sample of 120 ten-year-old children The lengths are summarised in the table below.
no gaps

(a) Use interpolation to estimate the median of this distribution.
(b) Calculate estimates for the mean and the standard deviation of these data
One measure of skewness is given by

$$
\text { Coefficient of skewness }=\frac{3(\text { mean }- \text { median })}{\text { standard deviation }}
$$

c) Evaluate this coefficient and comment on the skewness of these data.
Greg suggests that a normal distribution is a suitable model for the foot lengths of ten-year-old children.
(d) Using the value found in part (c), comment on Greg's suggestion, giving a reason for your answer

b) $₹ f x=2055.5$
if $x^{2}=36500.25$
$\bar{x}=\frac{\sum f x}{n}=17.13 \quad \sigma=\sqrt{\frac{36500 \cdot 25}{120}-17.13^{2}}=3.28$

## 

5. The weight, $w$ grams, and the length, $l \mathrm{~mm}$, of 10 randomly selected newborn turtles are given in the table below

| $l$ | 49.0 | 52.0 | 53.0 | 54.5 | 54.1 | 53.4 | 50.0 | 51.6 | 49.5 | 51.2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $w$ | 29 | 32 | 34 | 39 | 38 | 35 | 30 | 31 | 29 | 30 |

(You may use $\mathrm{S}_{u}=33.381 \quad \mathrm{~S}_{\mathrm{w} t}=59.99 \quad \mathrm{~S}_{\mathrm{mv}}=120.1$ )
(a) Find the equation of the regression line of $w$ on $l$ in the form $w=a+b l$.
(b) Use your regression line to estimate the weight of a newborn turtle of length 60 mm .
(c) Comment on the reliability of your estimate giving a reason for your answer.

| $\omega$ | $=a+b l$ |
| ---: | :--- |
| $\psi y$ | $=a+b x$ |$\quad \omega=y \quad l=x$

$b=\frac{S x y}{S x x}=\frac{\text { Sew }}{S \ell l}=\frac{S 9.99}{33.381}=1.797$
$a=\bar{y}-b \bar{x} \Rightarrow a=\bar{\omega}-b \bar{l}$
$\bar{\omega}=\frac{q \omega}{n}=\frac{518.3}{10}=51.83$
$\bar{I}=\frac{2 l}{n}=\frac{32.7}{10}=3.27$
$a=51.83-1.797 \times 3.27=-60.445$
$\omega=-60.44 S+1.797 l$
b) $l=60 \quad \omega=47.37 \mathrm{Sy}$
c) Unreliable, no evidence to support this since max length in our data was 54.5 cm
C) Shew $=\frac{3(17.13-17.14)}{3.28}=-0.009$

Symmetrical shew
d)
yes normal distribution seems appropric Since symmetrical skew mean $\simeq$ median $95 \%$ of data should-lie within $\bar{x} \pm 20$ $17.13 \pm 6.56$ ( $10.6-23.7$ ) which seems to be the case.
6. The discrete random variable $X$ has probability function

$$
\mathrm{P}(X=x)=\left\{\begin{array}{cl}
a(3-x) & x=0,1,2 \\
b & x=3
\end{array}\right.
$$

(a) Find $\mathrm{P}(X=2)$ and complete the table below.

| $x$ | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | $3 a$ | $2 a$ | $\mathbf{a}$ | $b$ |$\quad$| $\mathrm{E}(\boldsymbol{x})=0+2 a+2 a+3 b$ |
| :---: |

Given that $\mathrm{E}(X)=1.6$

$$
\Rightarrow 4 a+3 b=1.6
$$

(b) Find the value of $a$ and the value of $b$.
(5)

## Find

(c) $\mathrm{P}(0.5<x<3)$,
$\begin{array}{lllll}x^{2} & 0 & 1 & 4 & 9\end{array}$
$p \quad 0.3 \quad 0.20 .10 .4$
(d) $\mathrm{E}(3 X-2)$.

$$
\begin{equation*}
\epsilon\left(x^{2}\right)=0+0.2+0.4+3.6 \tag{2}
\end{equation*}
$$

$$
\begin{equation*}
\epsilon\left(x^{2}\right)=4.2 \tag{2}
\end{equation*}
$$

(e) Show that the $\operatorname{Var}(X)=1.64$
(f) Calculate $\operatorname{Var}(3 X-2)$.
b) $18 a+3 b=3$ -
$4 a+3 b=1.6$
$14 a=1.4 \Rightarrow a=0.1 \Rightarrow b=0.4$
c) $P(0.5<x<3)=P(1)+P(2)=3 a=0.3$
d) $\epsilon(3 x-2)=3 \epsilon(x)-2=3 \times 1.6-2=2.8$
e) $v(x)=\epsilon\left(x^{2}\right)-\epsilon(x)^{2}=4.2-1.6^{2}=1.64$
f) $v(3 x-2)=3^{2} \times v(x)=14.76$

8. The lifetimes of bulbs used in a lamp are normally distributed

A company $X$ sells bulbs with a mean lifetime of 850 hours and a standard deviation of 50 hours.
(a) Find the probability of a bulb, from company $X$, having a lifetime of less than 830 hours.
(b) In a box of 500 bulbs, from company $X$, find the expected number having a lifting of less than 830 hours

A rival company $Y$ sells bulbs with a mean lifetime of 860 hours and $20 \%$ of these bulbs have a lifetime of less than 818 hours.
(c) Find the standard deviation of the lifetimes of bulbs from company $Y$.

Both companies sell the bulbs for the same price
(d) State which company you would recommend. Give reasons for your answer
(2)
a) $P(x<830) \Rightarrow P\left(2<\frac{830-850}{50}\right)=P(2<-0.4)$ $=1-\phi(0.4)=0.3446$
b) $0.3446 \times 500=172.3 \quad 172$ bulbs
c) $P(x<818)=0.2 \Rightarrow P\left(z<\frac{-42}{\sigma}\right)=0.2$

d) $Y$ since it has a higher mean and the standard deviation is almost identical
$\qquad$
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